Total Pages : 8

UDHA–(Sem-I) Math (CC-2)

2017

Full Marks : 70

Time : 3 hours

There are three Groups A, B & C. Group-A is compulsory comprising of 10 (Ten) objective type questions for 02 (Two) marks each. Group-B contains 08 (Eight) short answer type questions of which 04 (Four) have to be answered for 05 (Five) marks each. Group-C contains 04 (Four) questions of long answer type questions of which 02 (Two) have to be answered for 15 (Fifteen) marks each.

The figures in the right-hand margin indicate marks

Candidates are required to give their answers in their own words as far as practicable

GROUP–A

1. Choose the *correct* answer :

(*a*) The radius of curvature at the origin for the curve $r = a\sin n\theta$ is equal to :

(*i*) a

(*ii*) $na/2$

(*iii*) na

(*iv*) $2na$

(*b*) If u is a homogeneous function of x and y of degree n then the value of $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$ is equal to :

(*i*) nu

(*ii*) 0

(*iii*) n^2u

(*iv*) $n(n-1)u$

(*c*) If $f(x) = e^{ax}$ then $f^n(0)$ is equal to :

(*i*) 1

(*ii*) e^a

(*iii*) e

(*iv*) None of these.

(*d*) The number of real asymptotes of the curve $x^3 + y^3 = 3axy$ equals to :

(*i*) 1

(*ii*) 2

(*iii*) 3

(*iv*) 4

(*e*) If $r = f(\theta)$ is any curve, then the polar subtangent is equal to :

(*i*) $r^2 \dfrac{d\theta}{dr}$

(*ii*) $r^2 \dfrac{dr}{d\theta}$

(*iii*) $\dfrac{1}{r^2} \cdot \dfrac{d\theta}{dr}$

(*iv*) $\dfrac{dr}{d\theta}$

(*f*) Focus of the parabola $x^2 + 4ay = 0$ is equal to :

(*i*) $(0, a)$

(*ii*) $(0, -a)$

(*iii*) $(-a, 0)$

(*iv*) $(a, 0)$

(*g*) The eccentricity of the conic $2x^2 + 3y^2 = 6$ is equal to :

(*i*) 3

(*ii*) 1/3

(*iii*) 0

(*iv*) None of these.

(*h*) The length of the latus rectum of the curve $b^2x^2 - a^2y^2 = a^2b^2$ is equal to :

(*i*) $\dfrac{2b^2}{a}$

(*ii*) $\dfrac{2b}{a^2}$

(*iii*) $\dfrac{2a^2}{b}$

(*iv*) $\dfrac{4b^2}{a}$

(*i*) The centre of the conic

$$8x^2+6y^2-16x+12y+13=0$$

is equal to :

(*i*) $(1,-1)$

(*ii*) $(-1,1)$

(*iii*) $(-1,-1)$

(*iv*) $(1,1)$

(*j*) The polar equation of a parabola taking its focus as the pole and initial line as the *x*-axis is given by :

(*i*) $l=r\cos\theta$

(*ii*) $l=r(1\pm\cos\theta)$

(*iv*) None of these.

GROUP–B

2. State and prove Leibnitz's theorem on successive differentiation.

3. State and prove Euler's theorem on partial differentiation of a homogeneous function of two independent variables.

4. State and prove Maclaurine's theorem to expand $f(x)$.

5. (*a*) Evaluate $\lim\limits_{x\to 0} \sin x \log x$

(*b*) Evaluate $\lim\limits_{x\to\infty} \dfrac{x^n}{e^x}$; *n* being a +ve integer.

6. Write down the equation of the director circle of the ellipse $b^2x^2+a^2y^2=a^2b^2$. Show that the director circle of the ellipse $9x^2+16y^2=144$ is a circle of radius 5 units.

7. Find the equation of the polar of the point (α,β) w.r.t. the conic

$$ax^2+2hxy+by^2+2gx+2fy+c=0.$$

8. Show that the polar equation of a conic is $l/r=1+e\cos\theta$ when the focus of the conic is taken as pole and the negative direction of the axis of the conic is taken as the initial line.

9. Trace the parabola

$$9x^2 - 24xy + 16y^2 - 50x - 100y + 225 = 0.$$

GROUP–C

10. (*a*) Find y_n when $y = e^{ax}\cos bx$. 5

(*b*) If $y = a\sin mx + b\cos mx$, prove that

$$y_2 + m^2 y = 0.$$ 5

(*c*) Find the maximum value of $\frac{\log_e^x}{x}$ 5

11. (*a*) Find the first four non-zero terms in the expansion of sec x in ascending powers of x. 5

(*b*) If $u = x^2 - y^2, v = 2xy$, evaluate

$$\frac{\partial(u,v)}{\partial(x,y)} \& \frac{\partial(x,y)}{\partial(u,v)}.$$ 5

(*c*) If $u = f(y/x)$, show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 0$. 5

12. (*a*) Find the equation of the normal at the point (α, β) to the ellipse $b^2x^2 + a^2y^2 = a^2b^2$. 5

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(*b*) If e_1 and e_2 be the eccentricities of a hyperbola and its conjugate, show that $\frac{1}{e_1^2} + \frac{1}{e_2^2} = 1$. 5

(*c*) Obtain the pole of the line $3x + 4y + 5 = 0$ w.r.t. the conic $x^2 + 4xy + 4y^2 - 8x + 5 = 0$. 5

13. (*a*) If any change of axes without the change of the origin, the expression $ax^2 + 2hxy + by^2$ becomes $a_1x^2 + 2h_1xy + b_1y^2$, then prove that $a + b = a_1 + b_1$. 5

(*b*) Find the condition that the general equation of second degree in x and y may represent a parabola, an ellipse and a hyperbola. 5

(*c*) Derive the condition that $y = mx + c$ may touch the ellipse $b^2x^2 + a^2y^2 = a^2b^2$. 5

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