UDHA(Sem-I) —
Phy (CC – 1)

2019

Time 3 hours

Full Marks 70

Candidates are required to give their answers in their own words as far as practicable

The figures in the margin indicate full marks.

*Answer from **all** the Sections as directed*

Section – A
(Compulsory)
(Objective Type Questions)

1. Choose the correct answer from the following : 2×10 = 20

(a) The necessary and sufficient condition that the line integral $\int_t F.\,dr$ for every closed C vanishes is :

(i) $\nabla \times F = 0$

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(ii) $\nabla \cdot F = 0$

(iii) $\times F \neq 0$

(iv) $\nabla \cdot F \neq 0$

(b) The spherical coordinate system is :

(i) Orthogonal

(ii) Coplanar

(iii) Non-coplanar

(iv) Non-orthogonal

(c) The value of $\iint_s (yzdydz + zxdzdx + xydxdy)$, where S is the surface of a unit sphere $x^2 + y^2 + z^2 = 1$ is :

(i) 0

(ii) 4π

(iii) $\frac{4\pi}{3}$

(iv) 10π

(d) If $f = \tan^{-1}(y / x)$ then $\nabla \cdot (\nabla F)$ is equal to :

(i) 1

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(ii) – 1

(iii) 0

(iv) 2

(e) The particular integral (PI) of $(D^2 + 4)y = \cos 2x$ is

(i) $\frac{1}{2}\sin 2x$

(ii) $\frac{1}{2}x\sin 2x$

(iii) $\frac{1}{4}x\sin 2x$

(iv) $\frac{1}{2}x\cos 2x$

(f) The complementray function (CF) of $y'' - 2y' + y = xe^x \sin x$ is

(i) $C_1e^x + C_2e^{-x}$

(ii) $(C_1x + C_2)e^x$

(iii) Both of these

(iv) None of these

(g) The differential equation $(x + x^8 + ay^2)\,dx + (y^8 - y + bxy)dy = 0$ is exact if

(i) $b = 2a$

(ii) $a = b$

(iii) $a \neq 2b$

(iv) $a = 1, b = 3$

(h) The solution of $\frac{d^2x}{dt^2} + n^2x = 0$ is

(i) $x = A\cos(nt + \alpha)$

(ii) $x = nt + \alpha$

(iii) $x = t$

(iv) $x^2 = \alpha$

(i) The degree of the differential equation

$$\left(\frac{d^2y}{dx^2}\right)^2 + x\left(\frac{dy}{dx}\right)^5 x^2y = 0 \text{ is}$$

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(i) 2

(ii) 0

(iii) 3

(iv) 5

(j) By the method of undetermined coefficient y_p of $y'' + 3y' + 2y = 12x^2$ is of the form .

(i) $a + bx + cx^2$

(ii) $a + bx$

(iii) $ax + bx^2 + cx^3$

(iv) None of these

Section – B

(Short-answer Type Questions)

Answer any **four** questions of the following : 5×4 = 20

2. Find $\nabla\phi$ if (a) $\phi = ln\,|r|$ and (b) $\phi = \frac{1}{r}$.

3. Solve $\frac{dy}{dx} = \sin(x + y) + \cos(x + y)$.

4. Solve $\left(1 + e^{\frac{x}{y}}\right) dx + e^{\frac{x}{y}} \left(1 - \frac{x}{y}\right) dy = 0$.

5. What is exact differential equation. Solve $(y^2 e^{xy^2} + 4x^3)\,dx + \left(2xye^{xy^2} - 3y^2\right) dy = 0$.

6. Solve the differential equation $Ri + L\frac{di}{dt} = E \sin \omega t$ and plot the solution.

7. Find the complete solution of $y'' - 2y' + 2y = x + e^x \cos x$

8. Prove $\nabla \times (\nabla \times F) = \nabla(\nabla \cdot F) - \nabla^2 F$

9. What is Cylindrical co-ordinate system ? How it is related with Cartesian coordinate system https://www.jharkhandstudy.com

Section – C

(Long-answer Type Questions)

Answer any **two** questions.

10. (a) State and prove Gauss' Divergence theorem 10

(b) Verify divergence theorem for $F = (x^2 - yz)\hat{i} + (y^2 - zx)\hat{j} + (z^2 - xy)\hat{k}$, taken over rectangular parallelopiped $0 \le x \le a$, $0 \le y \le b$ and $0 \le z \le c$ 5

11. (a) State and prove Green's theorem in a plane. 8

(b) Apply Green's theorem to evaluate $\int_C [(2x^2 - y^2)dx + (x^2 + y^2)\,dy]$, where C is boundary of the area enclosed by x-axis and the upper half of the circle $x^2 + y^2 = a^2$. 7

12. (a) Define orthogonal curvilinear coordinate and its transformation equations.

(b) Derive the Laplacian in spherical co-ordinate system 5+10 = 15

13. (a) Define (i) Gradient of a scalar field, (ii) Divergence and (iii) Curl of a vector field with their geometrical interpretations. 10

(b) Verify Stoke's theorem for $A = (2x - y)\hat{i} - yz^2\hat{j} - y^2z\hat{k})$, where S is the upper half surface of the sphere $x^2 + y^2 + z^2 = 1$ and C is its boundary. 5