UDHB(Sem-IV) — M
(CC – 9)

2019

Time : 3 hours

Full Marks : 70

Candidates are required to give their answers in their own words as far as practicable.

The figures in the margin indicate full marks.

*Answer **all** the Groups as directed.*

Group – A
(Compulsory)

1. Write down the correct answers of **all** questions from the available four options : 2×10 = 20

(a) $\int_0^\infty e^{-x}dx$ is :

(i) Convergent
(ii) Divergent
(iii) Uniformly convergent
(iv) None of these

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(b) Gauss theorem of divergence establishes relation between :

(i) Line integral and volume integral
(ii) Surface integral and volume integ⁻ 2/8
(iii) Line integral and surface integral
(iv) None of these

(c) If $\int_a^\infty |f(x)|\,dx$ converges, then $\int_a^\infty f(x)dx$:

(i) Also converges
(ii) Diverges
(iii) Not defined
(iv) None of these

(d) If P(x, y) and Q(x, y) are continuous functions having continuous partial derivatives in both variables over D, ('C' is traversed in positive or anti-clockwise direction) then,

$\int_C (Pdx + Qdy)$ is equal to :

(i) $\iint_D \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right)dx\,dy$

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(ii) $\iint_D \left(\frac{\partial Q}{\partial x} + \frac{\partial P}{\partial y}\right) dx\, dy$

(iii) $\iint_D \left(\frac{\partial^2 Q}{\partial x^2} + \frac{\partial^2 P}{\partial y^2}\right) dx\, dy$

(iv) None of these

(e) If x, y, z are all positive such that $h_1 \le x + y + z \le h_2$, then :

$\iiint F(x+y+z)\, x^{\ell-1} y^{m-1} z^{n-1}\, dx\, dy\, dz$ is equal to :

(i) $\frac{\Gamma(l)\,\Gamma(m)\,\Gamma(n)}{\Gamma(l+m+n)} \int_{h_1}^{h_2} F(h).h^{l+m+n-1}\, dh$

(ii) $\frac{\Gamma(l)+\Gamma(m)+\Gamma(n)}{\Gamma(l+m+n)} \int_{h_1}^{h_2} F(h).h^{l+m+n-1}\, dh$

(iii) $\frac{\Gamma(l+m+n)}{\Gamma(l)\,\Gamma(m)\,\Gamma(n)} \int_{h_1}^{h_2} F(h).h^{l+m+n-1}\, dh$

(iv) None of these

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(f) The additive inverse of an element 'a' of a ring R is

(i) Unique

(ii) Not unique

(iii) Depends on different conditions

(iv) None of these

(g) If a, b ∈ R the equation a + x = b has in R a unique solution given by :

(i) x = b – a

(ii) x = b + a

(iii) x = ab

(iv) None of these

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(h) If F is a field, then its only ideals are :

(i) Prime Ideals

(ii) Maximal Ideals

(iii) (0) and F

(iv) None of these

(i) A ring having no proper ideals is called :

(i) A commutative ring

(ii) A simple ring

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(iii) A commutative and simple ring

(iv) None of these

(j) Let R be a ring. if R is commutative, then R[x] is

(i) Commutative

(ii) Non-commutative

(iii) No relation between R and R[x]

(iv) None of these

Group – B

Answer any four of the following : 5×4 = 20

2. Show that $\int_1^2 \frac{dx}{(x+1)\sqrt{x^2+1}}$ is convergent.

3. Show that : 5/8

$$\int_0^\infty \frac{\tan^{-1}(ax) - \tan^{-1}(bx)}{x}\,dx = \frac{\pi}{2}\log\left(\frac{a}{b}\right); 0 < b < a$$

4. Apply Stoke's theorem to calculate

$$\int_c (4ydx + 2zdy + 6ydz)$$

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where 'c' is the curve of intersection of $x^2 + y^2 + z^2 = 6z$ and $z = x + 3$.

5. Show that $\iint_s (axdydz + bydzdx + czdxdy) = \frac{4}{3}\pi(a + b + c)$, where 's' is the surface of the sphere $x^2 + y^2 + z^2 = 1$.

6. Prove that the set of even integers is a commutative ring under ordinary addition and multiplication but has no unity element.

7. Show that the intersection of two ideals of a ring R is an ideal of R. https://www.jharkhandstudy.com

8. Prove that the ring of polynomials over a field is an Eucldeam ring.

9. Prove that every homomorphic image of a commutative ring is commutative ring.

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Group – C

Answer any two of the following

10. (a) If ϕ is continuous in $]0, \alpha[$ and $\underset{x \to 0}{\text{Lt}}\, \phi(x) = \phi_0$ and $\underset{x \to \infty}{\text{Lt}}\, \phi(x) = \phi_1$ then show that

$$\int_0^\infty \frac{\phi(ax) - \phi(bx)}{x} dx = (\phi_0 - \phi_1) \log \frac{b}{a}$$ 8

(b) Show that the integral $\int_0^\infty e^{-\alpha x} \frac{\sin x}{x} dx$ is convergent when $\alpha \geq 0$ 7

11. (a) State and prove Stoke's theorem to establish relation between line integral and surface integral. 7

(b) To prove that

$$\iiint F(x+y+z) \cdot x^{l-1} \cdot y^{m-1} \cdot z^{n-1}\, dx\, dy\, dz = \frac{\Gamma(l)\,\Gamma(m)\,\Gamma(n)}{\Gamma(l+m+n)} \int_{h_1}^{h_2} F(u) \cdot u^{l+m+n-1} du$$

where, $h_1 < x + y + z < h_2$ 8

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12. (a) State and prove Fundamental Theorem on Homomorphism of Rings 8

(b) Let I be the set of integers and let the addition and multiplication be defined as follows

$a \oplus b = a + b - 1$, $a \otimes b = a + b - ab$, $a, b \in I$. Prove that the set I is a commutative ring. 7

13. (a) Show that a field has no proper ideals. 5

(b) Show that S is an ideal of S + T where S is any ideal of R and T is any subring of R 5

(c) If R is an arbitrary ring and R′ is the set of constant polynomials in R[x], then R′ is isomorphic to R 5

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