2019

Time : 3 hours

Full Marks : 80

Pass Marks : 36

Candidates are required to give their answers in their own words as far as practicable.

The figures in the margin indicate full marks.

Answer from both the Sections as directed.

Section – I

(Compulsory)

1. Choose the correct alternatives from the following multiple choice objective questions : 2×8 = 16

(a) If $y = x^G$ then y_{n+1} is equal to :

(i) 1

(ii) 0

(iii) $\lfloor n$

(iv) None of these

(b) If $u = x^2 + y^2 + z^2$, then $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z}$ is equal to :

(i) u

(ii) 2u

(iii) 3u

(iv) None of these

(c) In a polar curve the length of the subnormal is :

(i) $\frac{dr}{d\theta}$

(ii) $\frac{d\theta}{dr}$

(iii) $\frac{r\,dr}{d\theta}$

(iv) None of these

(d) In a cartesian curve radius of curvature at any point (x, y) is equal to :

(i) $\frac{(1+y_2^2)^{3/2}}{y_1}$

(ii) $\frac{(1+y_1^2)^{3/2}}{y_2}$

(iii) $\frac{(1+y_1^2)^{3/3}}{y_2}$

(iv) None of these

(e) The centre of the circle $x^2 + y^2 - 2x - 4y - 10 = 0$ is :

(i) (1, 2)

(ii) (–1, – 2)

(iii) (–2, – 4)

(iv) None of these

(f) The radical axis of two circles $x^2 + y^2 + 3x + 5y + 4 = 0$ and $x^2 + y^2 + 5x + 3y + 4 = 0$ is equal to :

(i) $x + y = 0$

Unit – I

2. (a) State and prove Leibnitz theorem.

(b) If $\log y = \tan^{-1}x$ then prove that $(1 + x^2)\, y_{n+2} + (2nx + 2x - 1)y_{n+1} + n(n+1)y_n = 0$.

3 (a) If u is a homogeneous function of degree n then show that :

$$x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = n(n-1)u$$

(b) If $u = \sec^{-1}\frac{x^2+y^2}{x-y}$, prove that

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \cot u.$$

4 (a) To prove that $\frac{ds}{d\theta} = \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}$.

(b) Show that the tangent at (a, b) to the curve $(x/a)^3 + (y/b)^3 = 2$ is $x/a + y/b = 2$.

5. (a) Find the radius of curvature for the pedal curve $p = f(r)$.

(b) Find the radius of curvature of $x^2 + 2y^2 - x + y + 1 = 0$ at (2, 3).

Unit – II

6. (a) Find the equation of a circle cutting three circles $x^2 + y^2 + 2g_r x + 2f_r y + c_r = 0$ (where $r = 1, 2, 3$) orthogonally.

(b) Find the equation of the circle whose diameter is the common chord of the circles $x^2 + y^2 + 2x + 3y + 1 = 0$ and $x^2 + y^2 + 4x + 3y + 2 = 0$.

7. (a) Define Radical axis and show that the radical axis of two circles is a straight line perpendicular to the line joining the centres.

(b) Find the radical axis and the length of the common chord of the two circles $x^2 + y^2 + 3x + 5y + 4 = 0$ and $x^2 + y^2 + 5x + 3y + 4 = 0$.

8. Define parabola and obtain its equation in standard form.

9. (a) Find the equation of the tangent at the point 'α' of the conic $\ell/r = 1 + e\cos\theta$.

(b) If PSP′ and QSQ′ are two perpendicular focal chord of a conic. Prove that

$$\frac{1}{PP'} + \frac{1}{QQ'}$$ is constant.