

**2018**

Time : 3 hours

Full Marks : 80

Candidates are required to give their answers in their own words as far as practicable.

The figures in the margin indicate full marks.

Answer from both the Sections as directed.

**Section – I**

**(Compulsory)**

1. Answer all questions : 2×8 = 16

(a) Evaluate :

$$\int \frac{dx}{x^2 - a^2}$$

(b) Show that :

$$\int_0^a f(x) dx = \int_0^a f(a-x) dx$$

(c) Prove that :

$$\int_0^{\pi/2} \sin^4 x \cos^4 x dx = \frac{\pi}{32}$$

(d) Evaluate :

$$\int_0^{\pi/2} \frac{\sin x}{\sin x + \cos x} dx$$

(e) Define direction cosines of a line. If  $(l, m, n)$  be the direction cosines of any line, then what is the value of  $l^2 + m^2 + n^2$  ?

(f) What is the direction cosines of the normal to the plane  $x + y + z = 1$  ?

(g) Find the co-ordinates of the point of intersection of the line :

$$\frac{x-1}{2} = \frac{y-2}{-3} = \frac{z+3}{4}$$

with the plane  $2x - 2y - z = 7$ .

(h) Find the length of shortest distance between the lines :

$$\frac{x-2}{1} = \frac{y-1}{-2} = \frac{z-6}{1} \quad \text{and} \quad \frac{x+3}{7} = \frac{y+3}{-6} = \frac{z+3}{1}$$

**Section – II**

Answer four questions selecting at least one from each Group.

**Group – A**

2. (a) Evaluate : 8

(i)  $\int e^{ax} \sin bx \, dx$

(ii)  $\int \frac{x \sin^{-1} x}{\sqrt{1-x^2}} \, dx$

(b) Evaluate : 8

(i)  $\int (x+2)\sqrt{2x+1} \, dx$

(ii)  $\int \frac{xdx}{(x-a)(x-b)}$

3. (a) Prove : 8

$$\int_{-a}^a f(x)dx = \begin{cases} 2 \int_0^a f(x) \, dx, & \text{if } f(x) \text{ is even function} \\ 0, & \text{if } f(x) \text{ is odd function} \end{cases}$$

(b) Evaluate :

(i)  $\int_0^{\pi/2} \frac{\sin 2x}{\sin x + \cos x} \, dx$

(ii)  $\int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}} \, dx$

4. (a) If  $V_{m,n} = \int \cos^m x \cos nx \, dx$ , then show that : 8

$$V_{m,n} = \frac{\cos^m x \sin nx}{m+n} + \frac{m}{m+n} V_{m-1, n-1}$$

(b) Evaluate : 8

$\int_0^{\pi/2} \sin^4 x \cos^4 x \, dx$  by reduction formula

5. (a) Find the area of the loop of the curve  $x^3 + y^3 = 3axy$ . <https://www.jharkhandstudy.com> 8

(b) Find the area of the cardioid : 8  
 $r = a(1 + \cos\theta)$

**Group – B**

6. (a) Find the angle between the two lines whose direction cosines are  $(l_1, m_1, n_1)$  and  $(l_2, m_2, n_2)$ . 8

(b) If a line makes angles  $\alpha, \beta, \gamma, \delta$  with the four diagonals of a cube, then show that: 8

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma + \cos^2\delta = \frac{4}{3}.$$

7. (a) Find the condition of coplanarity of two lines

$$\frac{x-x_1}{l_1} = \frac{y-y_1}{m_1} = \frac{z-z_1}{n_1} \text{ and } \frac{x-x_2}{l_2} = \frac{y-y_2}{m_2} = \frac{z-z_2}{n_2}.$$

(b) Find the image of the point A(1, 3, 4) in the plane  $2x + z + 3 = y$ . 8

8. (a) A variable plane which remains at a constant distance  $3p$  from the origin and cuts co-ordinate axes at A, B, C. Find the locus of the centroid of the triangle ABC. 8

(b) Find the equation of the plane through the point (2, -3, 4) and parallel to the plane  $2x - 6y - 7z = 6$ . 8

9. (a) Find the magnitude and the equation of the shortest distance between two given straight lines. 8

(b) Find the distance of A(1, -2, 3) from the line PQ through P(2, -3, 5) which makes equal angles with the axes. 8



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