UG — Math
(C – 203)

2022(New)

Time : 3 hours

Full Marks : 80

Candidates are required to give their answers in their own words as far as practicable.

The figures in the margin indicate full marks.

Answer any five questions in which Q. No 1 is compulsory.

1. [A] Answer all questions : 1×8 = 8

(a) If $f(x) = \begin{cases} 1 \text{ if } x \text{ is a rational number} \\ 0 \text{ if } x \text{ is an irrational number} \end{cases}$,

then the value of (fof) $(\sqrt{3})$ is :

(i) 1

(ii) 0

(iii) 0 and 1 both

(iv) None of these

(b) The function $f : R \to R$, defined by $f(x) = e^x$ is :

(i) One – to – one

(ii) Onto

(iii) One – to – one and onto both

(iv) None of these

(c) A divergent sequence has :

(i) Infinite many limits

(ii) No limit

(iii) Unique limit

(iv) None of these

(d) The sequence $\left\{\frac{n^3 + 1}{n + 1}\right\}$ is :

(i) Decreasing

(ii) Increasing

(iii) Oscillatory

(iv) None of these

(e) The sequence $\left\{\frac{n + 1}{n}\right\}$ is :

(i) Increasing

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(ii) Oscillatory

(iii) Decreasing

(iv) None of these

(f) For the interval $I = (0, 1]$:

(i) 0 is a point of I

(ii) 0 is an isolated point of I

(iii) 0 is a limit point of I

(iv) None of these

(g) An infinite series Σt_n is said to be absolutely convergent if :

(i) $|\Sigma t_n|$ is convergent

(ii) $|\Sigma t_n|$ is convergent but Σt_n is divergent

(iii) $\Sigma |t_n|$ is convergent

(iv) None of these

(h) Which one is not an interval ?

(i) $(1, 2)$

(ii) $\left(\frac{1}{n}, n\right)$

(iii) $(3, \pi)$

(iv) None of these

[B] Answer any two questions of the following : $4 \times 2 = 8$

(a) Define neighbourhood of a point and limit point of a set in R.

(b) Define convergent and divergent sequence.

(c) Give examples of a monotonic increasing and a monotonic decreasing sequence.

(d) Give an example of a convergent and a divergent series.

2. Prove that the set of rational numbers is denumerable. 16

3. (a) Prove that the set of real numbers has Archimedean Property. 8

(b) For a, b $\in$ R, prove that the equation a + x = b has the unique solution x = (a) + b. 8

4. State and prove Cauchy's general principle of convergence of a sequence. 16

5. (a) Prove that a convergent sequence determines its limit uniquely. 8

(b) If for a sequence $\{a_n\}$; $S_n = 3(2^n - 1)$, find first three terms of the sequence. 8

6. State and prove Comparison test for a series of positive terms. 16

7. (a) State and prove D' Alembert's ratio test. 8

(b) Test for convergency of the series whose general term is $x^n . n$. 8

8. (a) Prove that the terms of a absolutely convergent series can be rearranged without affecting its convergence. 8

(b) Derange the series $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \frac{1}{6} + \cdots\cdots$ so that its sum is zero. 8

9. Write notes on any **two** of the following : 8×2 = 16

(a) Countable and uncountable sets

(b) Bounds of a sequence

(c) Convergency and divergency of a series

(d) Semi convergent and absolutely convergent series